PSI-PR-95-5

A comment on the anomalous $Zt\bar{t}$ couplings and the $Z \to b\bar{b}$ decay

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Abstract

By reanalyzing the influence of the anomalous $Z\bar{t}t$ couplings on the $Z \to b\bar{b}$ decay process, we pointed out the ambiguity in the conventional treatment of the effective Lagrangian approach, because of the possible existence of large contributions given by constant terms beyond the leading cutoff dependent term.

* supported by the "Ministerio de Education y Ciencia de Espana"

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The remarkable experimental result from LEP at CERN on the $Z \rightarrow b\bar{b}$ decay indicates that [1],

$$R_b = \frac{\Gamma_b}{\Gamma_{hadrons}} = 0.2192 \pm 0.0018 \ , \tag{1}$$

which is about 3σ away from the theoretical value predicted by the standard model [2], for the supported value of the top quark mass, $m_t = 174 \pm 20$GeV [3]. Intuitively, one may think that such a heavy top quark mass at the order of the Fermi scale may be useful in probing the electroweak symmetry breaking sector. As discussed in ref. [4], if the electroweak symmetry is broken dynamically one would expect, in general, the violation of the fermion–gauge boson coupling universality. The anomalous fermion–gauge boson coupling may naturally be expected at the order of $\sim \sqrt{m_i m_j}/v$ [5, 6], where i, j refer to the two fermions involved. Therefore the most important deviation from the standard model may be reflected in the anomalous $Zt\bar{t}$ couplings, which are characterized by two parameters, κ_L and κ_R, corresponding to the anomalous Z coupling to the left–hand current and the right–hand current, respectively. The observed discrepancy between the experimental value of R_b and the standard model prediction may therefore be explained by the nonvanishing of these anomalous couplings [5, 6, 7].

The effective $Zt\bar{t}$ Lagrangian obtained from a gauge invariant non-linear chiral Lagrangian for the Goldstone boson–top quark interactions, in the unitary gauge, can simply be written as,

$$L = \frac{g}{2c}\bar{t}\gamma_\mu \{P_L(1+\kappa_L) + P_R\kappa_R - \frac{8}{3}s^2\} t Z^\mu \ . \tag{2}$$

In order to read off the physical meaning of the non-renormalizable Lagrangian, it is appropriate to replace the divergence by a physical cutoff which is regarded as the scale of new physics, in a gauge invariant way. Following previous calculations we use dimensional regularization scheme that is to interpret $1/\epsilon$ by $\ln(\Lambda)$[8].

The physical observables at Z threshold can be quoted in four quantities, $\epsilon_{1,2,3}$ and δ_{bV} (for the definitions of these parameters, see ref. [9], [10]). The expressions of these quantities as functions of the anomalous $Zt\bar{t}$ couplings and the cutoff Λ were previously obtained, in the leading term approximation of the m_t^2 expansions, with the constant terms beside the $\ln\Lambda$ term systematically neglected. Using the new LEP data (see ref. [14], for the measure of δ_{bV} see ref. [12])

$$-2.4 \times 10^{-3} < \epsilon_1 < 2 \times 10^{-3} \ , \tag{3}$$

$$-5.8 \times 10^{-3} < \epsilon_2 < 4.8 \times 10^{-3} \ , \tag{4}$$

$$-5.2 \times 10^{-3} < \epsilon_3 < 0.8 \times 10^{-3} \ , \tag{5}$$

$$30.4 \times 10^{-3} < \delta_{bV} < 52.4 \times 10^{-3} \ . \tag{6}$$

we redraw the 1σ plot as shown in fig. 1. As can be seen from fig. 1, the constraints on κ_L and κ_R from the experimental values of the four observables are in severe disagreement with each other. In the effective Lagrangian approach to exploring the electroweak physics, one should in general write down the whole set of the dimension six operators, including the anomalous couplings among the gauge bosons (induced from the Goldstone boson–gauge boson interactions in the unitary gauge) [11] and the anomalous $Wt\bar{b}$ couplings [7]. Both the two types of anomalous couplings contribute to the epsilon quantities. Assuming a CP conserving effective interaction, in the former case there are 9 parameters while in the later case there are two, κ_L^{CC} and κ_R^{CC}. Including more parameters in the 1σ plot one expect that there can be a region in the parameter space fitting the experimental values. If one restricts to the anomalous gauge boson couplings only, it is shown [12] that influence on δ_{bV} comes essentially from only one operator in the effective Lagrangian. This approach, orthogonal to that of considering the anomalous $Zt\bar{t}$ couplings only, emphasizes different possible physics beyond the standard model as assuming the top quark playing no special role except that it has a large mass. Adding anomalous $Wt\bar{b}$ couplings, is helpful in solving the difficulties shown by fig. 1. However, the anomalous $Wt\bar{b}$ couplings will contribute to the $b \to s\gamma$ process by violating the GIM mechanism and may therefore receive strong constraints. A most general analysis, combining all the possible effects on the $Z \to b\bar{b}$ process, will however be less predictive.

One may conclude that the "minimal" fit in fig. 1 to the experimental results is inadequate, and therefore more parameters have to be added. However this simple approach is still possibly being self-consistent. The way is that in the previous calculation all the constant contributions (with respect to the term proportional to $\ln(\Lambda/m_t)$) induced by the anomalous couplings are neglected. In principle constant terms should also contribute to physical observables and in practice their contribution can be large numerically, as already known in chiral perturbation theory for hadron interactions. There is no a priori reason to neglect these effects. However the problem is that no unambiguous way exists in estimating them. In a cutoff dependent theory sub-leading terms are usually regularization procedure dependent. This can partly be considered as a reflection of the ambiguity in determining the explicit value of the cutoff parameter. In a renormalizable theory, all of these uncertainties are absorbed into the renormalized coupling constants which are identified as the experimentally observed values. Finite quantities, such as the anomalous magnetic moment of the electron, are cutoff independent and are therefore free of the ambiguity caused by the explicit dependence of the cutoff parameter[§]. Once the effective

[§]Terms which are vanishing when sending the cutoff to infinity are also regularization procedure

theory is embedded into the underlying renormalizable theory the cutoff parameter is replaced by some heavy mass scale which are physical observables [8]. The constant terms in the results given by the effective Lagrangian approach will be shifted because there are also contributions of such kind from high energy sector which are practically unknown (As argued in ref. [8] only the coefficient of the $\ln \Lambda$ term can be fixed). From these terms obtained from the effective Lagrangian we may at best know something about the order of magnitude of these finite corrections.

In the present case, we list the full expressions of the the above epsilons, including the constant terms, in the leading order of m_t^2 expansions: $\epsilon_i = \epsilon_i^{SM} + \epsilon_i'$ and $\delta_{bV} = \delta_{bV}^{SM} + \delta_{bV}'$ (ϵ_i^{SM} and δ_{bV}^{SM} are the standard model contributions),

$$\epsilon_1' = \frac{3G_F m_t^2}{4\sqrt{2}\pi^2}[2(k_R - k_L) - (k_R - k_L)^2] \ln(\frac{\Lambda^2}{m_t^2}) \ , \tag{7}$$

$$\epsilon_2' = -\frac{G_F m_W^2}{\sqrt{2}\pi^2}[(\frac{1}{2}k_L + \frac{k_L^2 + k_R^2}{4}) \ln(\frac{\Lambda^2}{m_t^2}) + \frac{k_R - k_L}{2} - \frac{(k_R - k_L)^2}{4}] \ , \tag{8}$$

$$\epsilon_3' = \frac{G_F m_t^2}{2\sqrt{2}\pi^2}[(\frac{2}{3}k_R - \frac{1}{3}k_L - \frac{k_L^2 + k_R^2}{2}) \ln(\frac{\Lambda^2}{m_t^2}) + k_L - k_R + \frac{(k_L - k_R)^2}{2}] \ , \tag{9}$$

$$\delta_{bV}' = \frac{G_F m_t^2}{\sqrt{2}\pi^2}[(k_L - \frac{1}{4}k_R) \ln(\frac{\Lambda^2}{m_t^2}) + \frac{1}{2}k_L + \frac{1}{8}k_R] \ . \tag{10}$$

There are still ambiguities in the above expressions since a redefinition of the cutoff parameter will change the constant terms. We may remove the part in each constant term which is proportional to the coefficient of the corresponding $\ln(\Lambda/m_t)$ term. In other words, this part is being absorbed by the cutoff dependent term and the remaining quantity, by definition, is not influenced by the change of the cutoff parameter. Then we may rewrite the above expressions as:

$$\epsilon_1' = \frac{3G_F m_t^2}{4\sqrt{2}\pi^2}[2(k_R - k_L) - (k_R - k_L)^2] \ln(\frac{\Lambda^2}{m_t^2}) \ , \tag{11}$$

$$\epsilon_2' = -\frac{G_F m_W^2}{\sqrt{2}\pi^2}[(\frac{1}{2}k_L + \frac{k_L^2 + k_R^2}{4}) \ln(\frac{\Lambda^2}{m_t^2}) + \frac{k_R}{2} + \frac{k_R k_L}{2}] \ , \tag{12}$$

$$\begin{aligned} \epsilon_3' = \frac{G_F m_t^2}{2\sqrt{2}\pi^2}[(\frac{2}{3}k_R - \frac{1}{3}k_L - \frac{k_L^2 + k_R^2}{2}) \ln(\frac{\Lambda^2}{m_t^2}) \\ + \frac{2}{5}k_L + \frac{1}{5}k_R - \frac{2}{5}(k_L^2 + k_R^2) - k_L k_R] \ , \end{aligned} \tag{13}$$

$$\delta_{bV}' = \frac{G_F m_t^2}{\sqrt{2}\pi^2}[(k_L - \frac{1}{4}k_R) \ln(\frac{\Lambda^2}{m_t^2}) + \frac{4}{17}(\frac{1}{4}k_L + k_R)] \ . \tag{14}$$

dependent, even in renormalizable theories. This means if we consider the renormalizable theory as an "effective" theory, i.e., keeping the cutoff parameter finite, there are also ambiguities, at the $O(1/\ln \Lambda)$ or $O(1/\Lambda)$ level.

The effects of these additional terms can be seen by comparing fig. 1 with fig. 2. We find that their effects are large. In the present example, they improve impressively the discrepancies shown in fig. 1. Therefore it may still be possible to explain the 3σ discrepancies between the experimental values and the standard model prediction of R_b in terms of only two anomalous parameters. Certainly, we are not able to demonstrate this two parameter fit is valid when taking the constant contributions into account. What one can only conclude from the above example is that the conventional treatment to the effective Lagrangian approach to exploring the possible physics beyond the standard model suffers from ambiguities: the uncontrollable constant terms can be large in magnitude in some cases, and in the worst situation one may imagine (although not natural), can destroy results obtained by simply ignoring them.

The unambiguous way in exploring the possible anomalous effects, can be like that have been done in chiral perturbation theory[13]: including all possible terms at dimension six, cancelling all the divergence by counter terms, and bet that there are still predictions left. However since too many parameters are involved in the present case, and in general, we suspect that the useful informations can be obtained are rather limited.

We thank Fred Jegerlehner and Claudio Verzegnassi for helpful discussions.

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Fig.1: Contourplot of $\epsilon'_1, \epsilon'_2, \epsilon'_3, \delta'_{bV}$ including only the $\log\Lambda$ terms in function of k_L, k_R with the constraints of eqs. (3) to (6) and with $m_t = 174$ GeV, $\Lambda = 1$ TeV, $m_H = 300$ GeV. The interval of variation of the $\epsilon'_i, \delta'_{bV}$ parameters have been found imposing the constraints at 1 σ:

$$\epsilon_i'^{max} = (\bar{\epsilon}_i - \epsilon_i'^{SM}) + \epsilon_i^{max}$$

$$\epsilon_i'^{min} = (\bar{\epsilon}_i - \epsilon_i'^{SM}) + \epsilon_i^{min}$$

where $\bar{\epsilon}_i$ is the experimental mean value, ϵ_i^{SM} is the corresponding theoretical value, whereas $\epsilon_i^{max(min)}$ are the extremal values at 1 σ away from $\bar{\epsilon}_i$.

Fig.2: The same as before but with the $\epsilon'_i, \delta'_{bV}$ including also the finite contributions.